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An algorithm is presented for implementing a digital matched filter for periodic and windowed periodic radar waveforms. The algorithm is based on circular convolutional techniques. For moderate to high duty cycles d (radar "on" time to pulse repetition interval (PRI) ratio), the circular convolutional algorithm uses 50% fewer complex multiply operations (CMOPs) than does a fast convolution implementation. In fact, if BPRI is the radar waveform bandwidth-PRI product, then the circular convolutional technique is numerically more efficient if $d > 1/\sqrt{8\,\mathrm{BPRI}}$. Furthermore, the circular convolution technique is numerically more efficient when the pulse compression ratio $\rho > \sqrt{\mathrm{BPRI}/8}$.

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A Numerically Efficient Digital Matched Filter for Periodic and Windowed Periodic Radar Waveforms

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Radar Division

February ~~26~~, 1988

CONTENTS

A NUMERICALLY EFFICIENT DIGITAL MATCHED FILTER FOR PERIODIC AND WINDOWED PERIODIC RADAR WAVEFORMS

I. INTRODUCTION

A network whose frequency response function maximizes the output peak signal-to-mean noise power (S/N) ratio is called a matched filter. Almost all radar receivers are designed with the matched filter criteria. If $h(t)$ is the impulse response function of the matched radar receiver, $s(t)$ is the transmitted radar waveform, and the noise interference is white and additive, then it can be shown [1-3] that

$$h(t) = s^*(-t), \tag{1}$$

where * denotes the complex conjugate operation.

A digital matched receiver design is based on the same principle of maximizing (S/N). We sample the transmitted radar waveform at equal time intervals τ. Let $s_1, s_2, \ldots, s_N$ be the values of the sampled transmitted waveform where N is the number of sampled points as seen in Fig. 1. We set

$$\mathbf{s} = (s_1, s_2, \ldots, s_N)^T, \tag{2}$$

where T denotes the vector transpose operation. The digital receiver applies a weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_N)^T$ such that

$$y = \mathbf{w}^T \mathbf{s}. \tag{3}$$

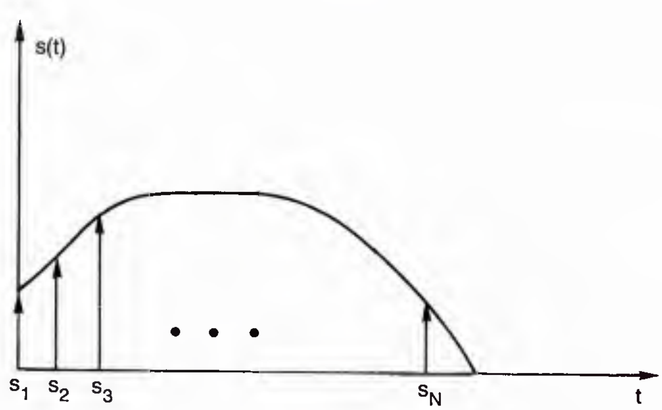

Fig. 1 — Sampled radar waveform

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It can be shown that (S/N) is maximized when

$$\mathbf{w} = \mathbf{s}^* . \tag{4}$$

The matched receiver structure can be represented as seen in Fig. 2. Here $x(n)$ is the received time-sampled input sequence consisting of signal plus noise. The received input sequence is digitally convolved with the conjugated time-reversed sequence of the vector s, denoted by $\tilde{\mathbf{s}}^*$, which results in a matched output sequence $y(n)$. Mathematically, this can be stated as

$$y(n) = \sum_{k=0}^{N-1} s_{k+1}^* x(n+k). \tag{5}$$

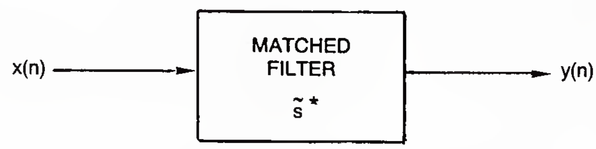

Fig. 2 — Simplified digital matched filter

Many surveillance radars employ a periodic waveform as illustrated in Fig. 3. The waveform is "on" for a given duty cycle and then turned "off" to receive the reflected echoes. The "on" portion of the waveform is subdivided into N cells where the nth cell has the value s_n. Each cell is τ seconds long, where τ is proportional to the range resolution cell. In fact, it can be shown [3] that if β is the bandwidth of the radar waveform, then $\tau \approx 1/\beta$. Let there be M range resolution cells in the pulse repetition interval (PRI). It can be shown that PRI $= M\tau$.

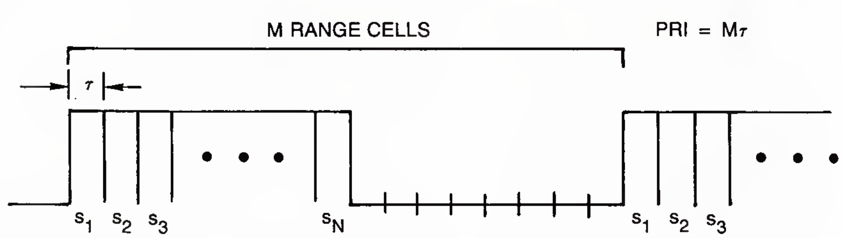

Fig. 3 — Periodic radar waveform

The digitally matched receiver of this periodic waveform can be implemented by using the linear convolution operation as given by Eq. (5) or it can be implemented by using fast convolution techniques [4]. In this report, we introduce a processing method (based on circular convolutional techniques) that is numerically more efficient than the fast convolution techniques when the duty cycle is not small. In fact, for 50% duty cycle waveforms, the technique uses half as many complex multiplication operations (CMOPs) as do the fast convolution techniques. The circular convolution algorithm is presented in Section II and is compared with a fast convolution technique in Section III.

Note that if the radar waveform pulse train is a long-windowed periodic function, then the circular convolution technique is also applicable with some (S/N) losses occurring at the leading and trailing edges of the received waveform.

II. MATCHED FILTER IMPLEMENTATION

In this section, we present a matched filter implementation based on circular convolution techniques. Consider the simplified digital receiver structure shown in Fig. 4. The return signal is $x(t)$ and is sampled each τ seconds. Each sample is successively stored in a shift register (SR) until M samples are taken. We match the filter to those M samples. After matching, the SR is reloaded with the next M samples of $x(t)$, matched, and so on. Note that when this implementation scheme is employed, in most cases we are matching the received data, $x_1, x_2, \ldots, x_M$ across two PRIs. For example, with no noise, $x_1 = s_m$, $x_2 = s_{m+1}, \ldots, x_{N-m+1} = s_N$, $x_{N-m+2} = 0$, $x_{N-m+3} = 0, \ldots, x_{M-m-3} = 0$, $x_{M-m-2} = s_1$, $x_{M-m-1} = s_2, \ldots x_M = s_{m-1}$, where m is related to referenced time delay or compressed range cells.

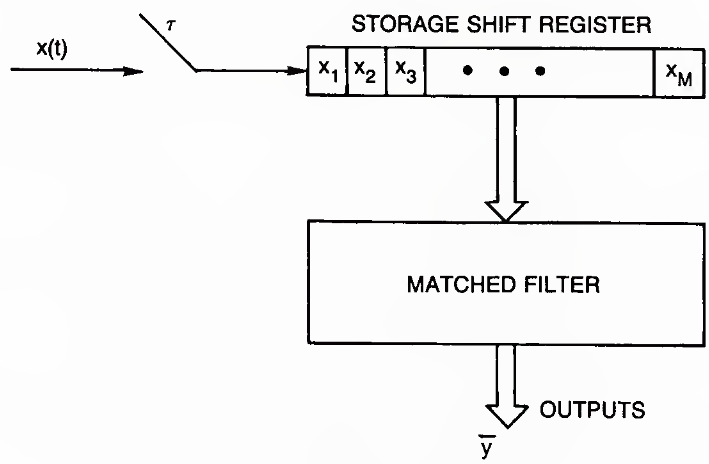

Fig. 4 — Simplified digital receiver

We designate the compressed range cells as $r_1, r_2, \ldots, r_M$. The matched filter output for the data contained in the SR for each range cell is

$$
\begin{aligned}
r_1 &: \quad y_1 = s_1^* x_1 + s_2^* x_2 + \ldots + s_N^* x_N \\
r_2 &: \quad y_2 = s_1^* x_2 + s_2^* x_3 + \ldots + s_N^* x_{N+1} \\
r_3 &: \quad y_3 = s_1^* x_3 + s_2^* x_4 + \ldots + s_N^* x_{N+2} \\
& \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
r_M &: \quad y_M = s_1^* x_M + s_2^* x_1 + \ldots + s_N^* x_{N-1},
\end{aligned}
\tag{6}
$$

where y_m, $m = 1, 2, \ldots, M$ represent the compressed output for each range cell.

Let

$$\mathbf{x} = (x_1, x_2, \ldots, x_M)^T \tag{7}$$

$$\mathbf{y} = (y_1, y_2, \ldots, y_M)^T \tag{8}$$

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & \ldots & s_N & 0 & \ldots & 0 \\ 0 & s_1 & s_2 & \ldots & s_{N-1} & s_N & \ldots & 0 \\ 0 & 0 & s_1 & \ldots & s_{N-2} & s_{N-1} & \ldots & 0 \\ . & . & . & \ldots & \ldots & \ldots & \ldots & . \\ . & . & . & \ldots & \ldots & \ldots & \ldots & . \\ . & . & . & \ldots & \ldots & \ldots & \ldots & . \\ s_2 & s_3 & s_4 & \ldots & 0 & 0 & \ldots & s_1 \end{bmatrix} \tag{9}$$

We can then show that

$$\mathbf{y} = S^* \mathbf{x}. \tag{10}$$

However S is a circular Toeplitz matrix [5] and can be written in the form

$$S = B \Lambda B^* , \tag{11}$$

where the Butler matrix

$$B = (\Gamma_M^{(m-1)(n-1)}); \qquad m, n = 1, 2, \ldots, M, \tag{12}$$

$$\Gamma_M = e^{-j\frac{2\pi}{M}},$$

the diagonal matrix

$$\Lambda = (\lambda_{mm}); \qquad m = 1, 2, \ldots, M, \tag{13}$$

and

$$\lambda_{mm} = \frac{1}{\sqrt{M}} \sum_{k=0}^{N-1} s_{k+1} \Gamma_M^{(m-1)k} \ ; \ m = 1, 2, \ldots, M. \tag{14}$$

Hence $\mathbf{y}$ can be rewritten as

$$\mathbf{y} = B^* \Lambda^* B \mathbf{x}. \tag{15}$$

As a result, the matched filter implementation can be configured as shown in Fig. 5. Note that this implementation is equivalent to a circular convolution [4].

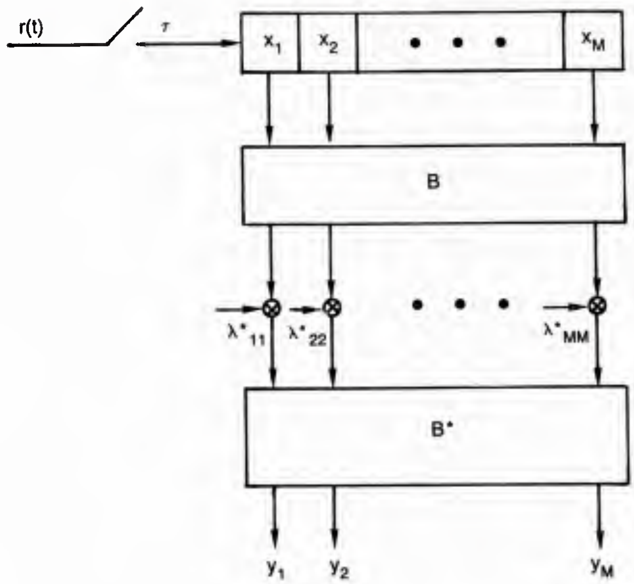

Fig. 5 — New matched filter implementation

An efficient implementation of this multiple channel matched filter is accomplished by using M-point fast Fourier transforms (FFTs) which is equivalent to multiplying by the B matrix; and by using M-point inverse FFTs, which is equivalent to multiplying by B^*. This implementation is shown in Fig. 6. The input data structure is as illustrated in Fig. 7, for $N = 8$ and $M = 16$. Note that the input data structure differs from that of the linear convolver in that the "matching" in most cases actually occurs over two adjacent pulses. In the example seen in Fig. 7, three subpulses s_6, s_7, s_8 in pulse 1 are inputted along with the five subpulses s_1, s_2, s_3, s_4, s_5 in pulse 2 into the matched filter (circular convolver implementation).

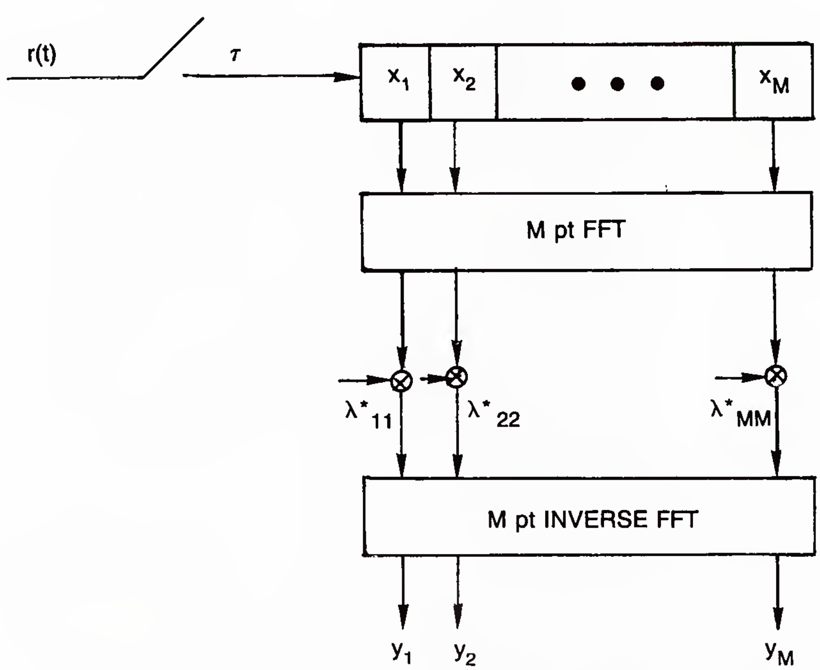

Fig. 6 — Efficient implementation of new matched filter

Doppler processing is also implementable in conjunction with using the circular convolver as a matched filter or pulse compressor as illustrated in Fig. 8. Here, we have inserted a Doppler filter bank before pulse compression (note that ideally the operations of Doppler filtering and pulse compression can be reversed with no differences in the respective output channels). The Doppler filter bank samples the input data stream $x(t)$ every PRI, which we set equal to T. We make the following definitions. Let $\bar{a}_l = (a_0^{(l)}, a_1^{(l)}, \ldots, a_{K-1}^{(l)})^T$ be the K Doppler coefficients on the lth Doppler filter. Let L be the number of Doppler filters and ϕ be the Doppler phase shift over one PRI of our desired signal. Thus the output s''_n of the lth Doppler filter related to the nth subpulse of the received desired signal is

$$s''_n = a_0^{(l)} s_n + a_1^{(l)} s_n e^{j\phi} + a_2^{(l)} s_n e^{j2\phi} + \ldots + a_{K-1}^{(l)} s_n \cdot e^{j(K-1)\phi} \tag{16}$$

$$= s_n \sum_{k=0}^{K-1} a_k^{(l)} e^{jk\phi}$$

$$= s_n f_l(\phi),$$

where we have defined

$$f_l(\phi) = \sum_{k=0}^{K-1} a_k^{(l)} e^{jk\phi} \tag{17}$$

to be the Doppler filter gain factor of the lth filter. Hence the output data stream of the desired signal through the lth filter looks as is shown in Fig. 9. Note that the input data stream for the desired signal is multiplied by the Doppler filter gain factor, which is range independent, and also that the basic periodic structure is retained. Thus this signal can be matched filtered by using the circular convolution procedure as if there were no Doppler processing present.

III. NUMERICAL EFFICIENCY COMPARISON

The total number of complex multiplications operations (CMOPs) per PRI to implement the circular convolution algorithm is tabulated below:

		CMOPs
M-pt inverse FFT	:	$.5M \log_2 M$
λ weighting	:	M
M-pt FFT	:	$.5M \log_2 M$
Total	:	$M \log_2 2M$.

We compare this with the number of CMOPs per PRI if the standard matched receiver (Fig. 2) is implemented.

In Fig. 2, $x(m)$, $m = 0, 1, \ldots,$ are consecutive samples of $x(t)$. This sequence is convolved with the matched finite impulse response filter, which has coefficients $s_1, \ldots, s_N$ to form the matched output sequence: $y(0)$, $y(1)$, ... If standard convolution is employed to generate a single output

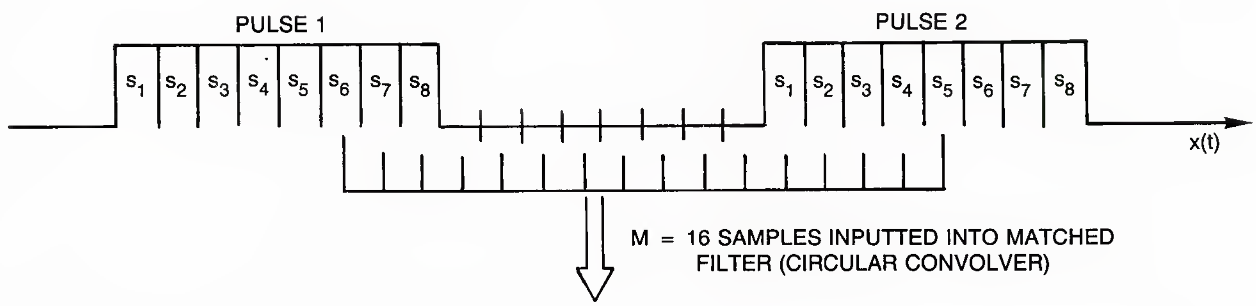

Fig. 7 — Input data structure into the circular convolver

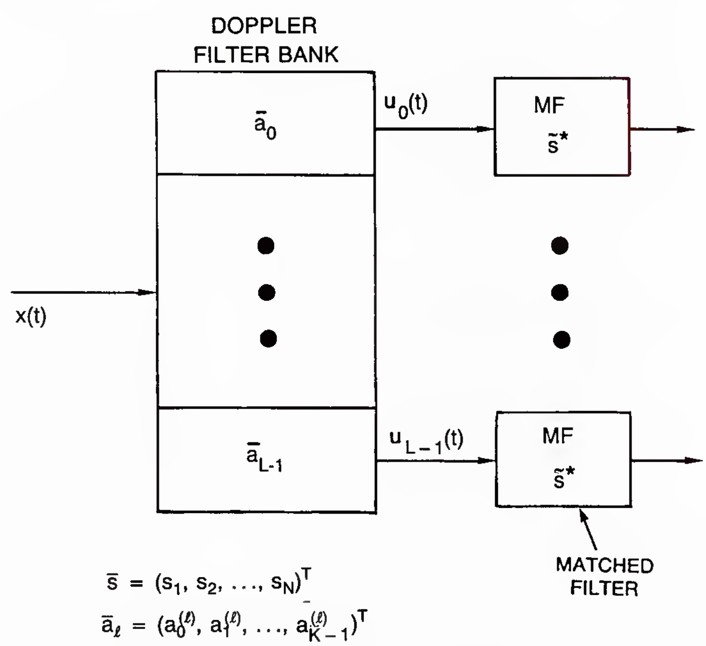

Fig. 8 — Doppler processing and matched filtering

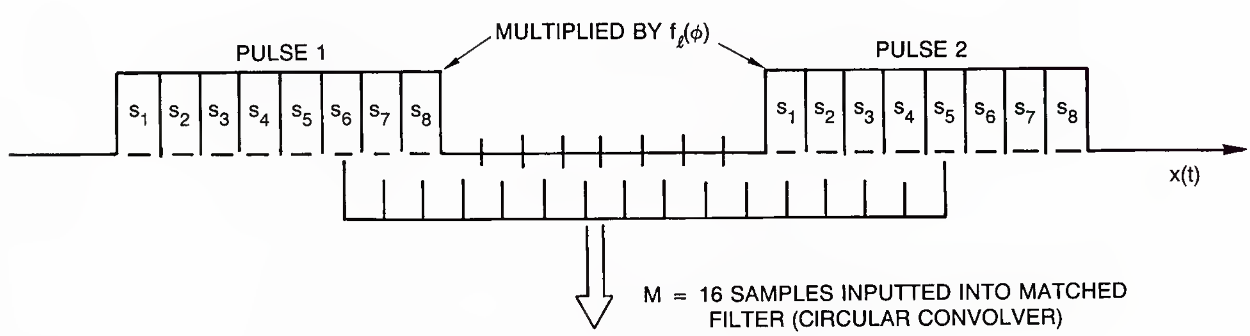

Fig. 9 — Input data structure after Doppler processing

point, N CMOPs are necessary (see Eq. (5)). Thus to generate M outputs (or all the matched outputs over a PRI), MN CMOPs are needed.

If fast convolution techniques are employed, then we can show that to generate N outputs (overlap and add technique [4]), then $2N \log_2 4N$ CMOPs are needed (see tabulation below).

		CMOPs
$2N$ FFT of N-pt input	:	$N \log_2 2N$
internal multiply by the DFT {s}	:	$2N$
$2N$ inverse FFT	:	$N \log_2 2N$
Total	:	$2N \log_2 4N$.

Hence to generate M points,

$$\frac{M}{N}(2N \log_2 4N) = 2M \log_2 4N \tag{18}$$

points are needed.

We list the number of CMOPs per PRI for each algorithm

Algorithm	CMOPs per PRI
New	$M \log_2 2M$
Fast Convolve	$M \log_2 16N^2$
Standard Convolve	MN

Thus if $2M < 16N^2$, then the circular convolutional matched filter is more efficient. For example, if the duty cycle of the waveform is 50%, then $M = 2N$. The new algorithm takes $2N \log_2 4N$ CMOPs, and the fast convolve algorithm takes $4N \log_2 4N$ CMOPs. Hence the new algorithm requires half as many CMOPs as the fast convolution algorithm. In fact, for $d = 1$ or 100% duty cycle waveform, the number of CMOPs of the circular convolution algorithm is slightly less than half of the fast convolution algorithm.

If we define the duty cycle d to be the "on" time divided by the PRI, then $d = N/M$. Also, if ρ is the pulse compression ratio, then $\rho = N$. If BPRI is the radar waveform bandwidth-PRI product that is equal to the number of range cells per PRI, then

$$N = dM = d\,\text{BPRI}. \tag{19}$$

Using these above definitions, we can show that

$$\frac{\text{no. COMPs using fast linear convolution}}{\text{no. CMOPs using circular convolution}} = \frac{4 + 2\log_2 \rho}{1 + \log_2 \text{BPRI}}. \tag{20}$$

In Fig. 10, we plot the ratio expressed in Eq. (20) as a function of the number of range cells per PRI, BPRI, and the pulse compression ratio ρ. Note that if the ratio given in Eq. (20) is greater than one,

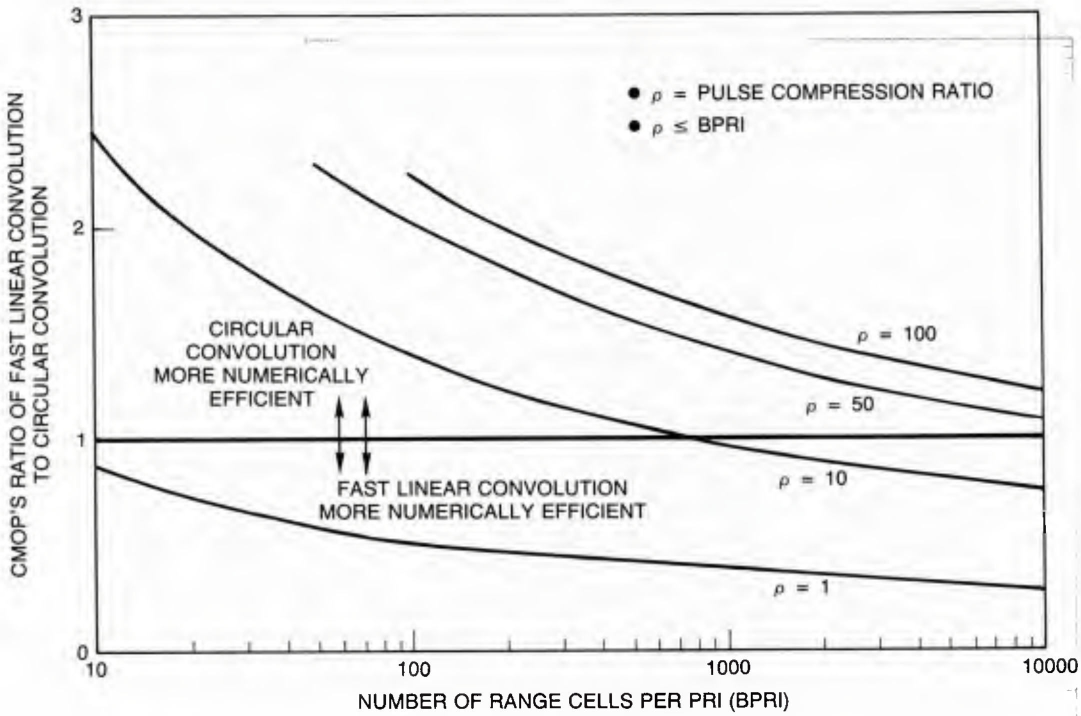

Fig. 10 — CMOP's ratio vs BPRI and ρ

then the circular convolution implementation is more numerically efficient than the fast linear convolution. Also observe that the circular convolution implementation tends to be more efficient for larger pulse compression ratios.

Using the above formulation, we can show that the circular convolution algorithm is more efficient when

$$d > \frac{1}{\sqrt{8 \cdot \mathrm{BPRI}}}, \text{ or } \rho > \sqrt{\frac{\mathrm{BPRI}}{8}}. \tag{21}$$

This is illustrated and plotted in Figs. 11 and 12. The region above the straight line indicates values of duty cycle (or pulse compression ratio) and BPRI where the circular convolution implementation is more numerically efficient than the fast linear convolution.

Finally, we note that there is a settling time of one extra PRI associated with using the circular convolutional matched filter as compared to using linear convolution. This is because the circular convolutional matched filter in most cases uses data from adjacent PRI. Hence, initially upon reception for a target at a given range, there is target data in one PRI but not in the preceding PRI which leads to an incomplete circular match. Furthermore, for finite length pulse trains, the last PRI will not be properly matched because the succeeding PRI has no target data. Also note that the absolute settling time of any matched receiver depends on the PRI (the possibility of second, third, etc. time around returns) and whether there is clutter processing.

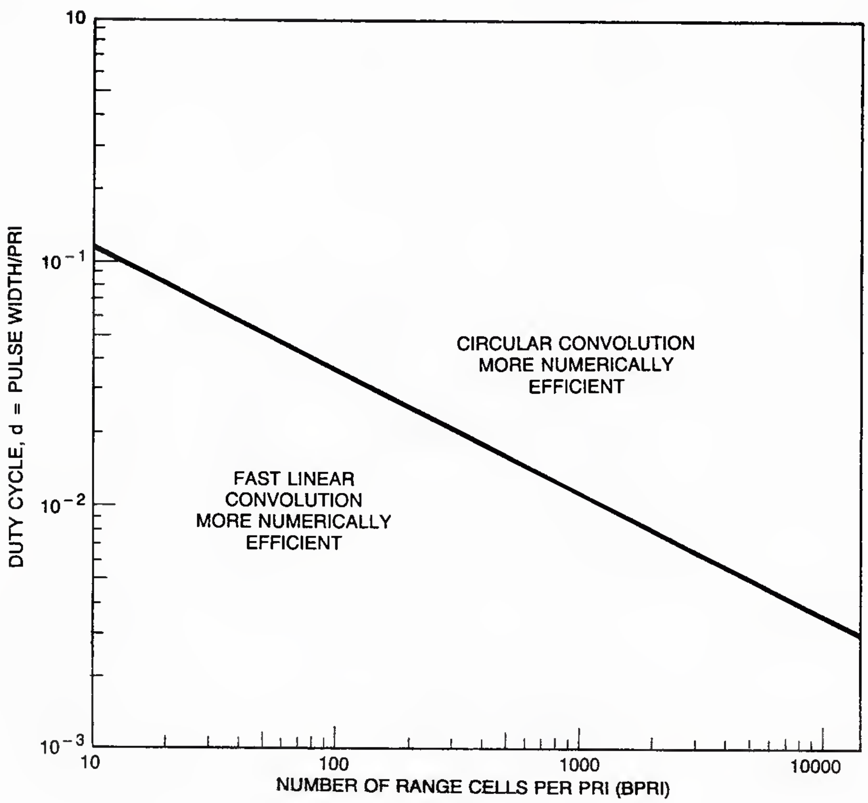

Fig. 11 — Duty cycle vs BPRI

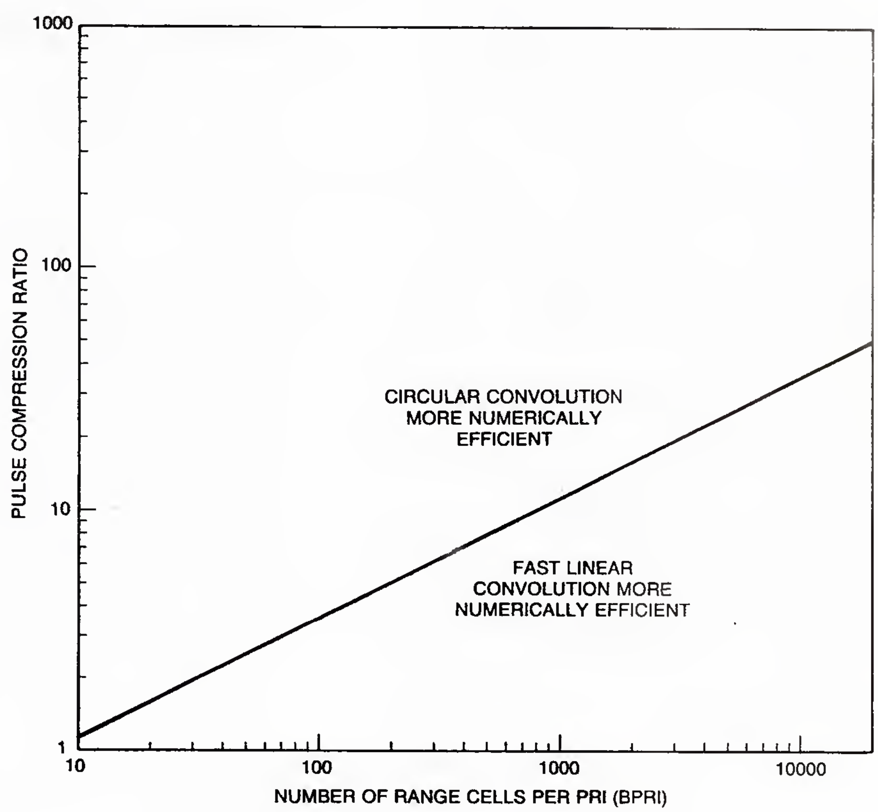

Fig. 12 — Pulse compression ratio vs BPRI

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